

Meaning, Reference and Modality A

Meanings, Worlds, Frames and Times

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Overview of the Course (in Keywords)

We talk about:

We formulate:

A	Meanings Frames	Possibilities Times
B	Necessity Existence	Identity [Indexicality]
C	Representation Information	Interpretation Inference

(Modal Propositional Logic)

(Modal Predicate Logic)

(Dynamic Semantics)

MEANINGS

Formal Semantics



- There is structure. (You MUST believe this.)
In reality, life, science, and in language.
- There is meaning. (You MUST believe this.)
In reality, life, science, and in language.
- There is structure in meaning. (Wishful Thinking?)
Compositionality, entailment, a prioricity,
- Formally Accounted for and Philosophically Reflected upon.

Three Tenets

Extensional Semantics The meaning (significance) of an expression resides in its reference (extension, denotation, truth, ...).

Einen Satz verstehen, heißt, wissen was der Fall ist, wenn er wahr ist.
[Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, Satz 4.024]

Intensional Semantics The meaning (sense) of a sentence resides in the determination of its reference (e.g., its truth-conditions).

[I]t seems to me in any case completely hopeless to try to divorce the idea of the meaning of a sentence from the idea of the information that the sentence can convey to a hearer or reader (...). [Jaakko Hintikka, 1969, "Semantics for Propositional Attitudes", in *Philosophical Logic*, J.W. Davis et al. (eds.)]

Dynamic Semantics The meaning (impact) of a sentence consists in its potential to update information.

Key Notions with Notation

	Satisfaction	Truth	Validity	Entailment
Ext.	$M, g \models \phi$	$M \models \phi$	$\models \phi$	$\Gamma \models \phi$
Int.	$M, w, g \models \phi$	$M, w \models \phi$	$(F) \models \phi$	$\Gamma \models \phi$
Dyn.	$M, \langle g, h \rangle \models \phi$	$M, g \models \phi$	$\models \phi$	$\vec{\gamma} \models \phi$

With:	M	a model of interpretation	g, h	assignments of values to variables
	w	a possible world		
	F	a modal frame		
	Γ	a set of formulas		

MODAL PROPOSITIONAL LOGIC

Principles of Extensional Semantics

- Terms denote objects:

“Fido” \mapsto *Fido*.

- Predicates have sets of objects as their extensions:

“Boys” \mapsto *the boys*;
“swim” \mapsto *the swimmers*.

- Sentences have as their reference a truth value:

“Fido is a boy.” \mapsto 0 iff *Fido* \notin *the boys*;
“Fido swims.” \mapsto 1 iff *Fido* \in *the swimmers*.

- Quantifiers are properties of or relations between sets:

“All” \mapsto \subseteq ;
“All boys swim.” \mapsto 1 iff *the boys* \subseteq *the swimmers*.

Meaning and Extensionality

Extensionality principles (c.f., GAMUT p. 5)

$$\begin{aligned} t = t' & \models \phi \leftrightarrow [t'/t]\phi; \\ \forall x(Ax \leftrightarrow A'x) & \models \phi \leftrightarrow [A'/A]\phi; \\ \psi \leftrightarrow \psi' & \models \phi \leftrightarrow [\psi'/\psi]\phi. \end{aligned}$$

Having the same (or no) reference is having the same (or no) meaning?

- *Identity.*
 - (1) Lois doesn't know that Leningrad is Saint Petersburg.
- *Existence.*
 - (2) Vulcan's orbit does not lie between that of Mars and Earth.
 - (3) Rumpelstiltskin does not exist.
- *Substitution.*
 - (4) It is obvious/necessary that 9 is 9. The number of planets is 9.
[?]So it is obvious/necessary that the number of planets is 9.

Acknowledging the Possibility of other Possibilities

- (5) It is raining, *in France*.
 - (6) You are a master, *one day*.
 - (7) (A:) There *are* goblins.
(B:) Sure. *In YOUR universe*.
 - (8) *In my dreams* there were 12 students, including Rumpelstiltskin.
 - (9) *According to Tugba*, there are possibly 55 students.
 - (10) *If you respect the law* your bicycle carries lights.
- Propositions that are false (/true) could have been true (/false).
 - A proposition can be true relative to one, and not another, possibility.
 - A proposition also can be possible relative to one, and not to the other.

Wittgenstein's Logical Logical Space

p	q	r	s	ϕ	...	ψ	$\uparrow(\phi, \psi)$...	
1	1	1	1	x	...	y	$1 - xy$...	w_1
1	1	1	0	w_2
1	1	0	1	w_3
1	1	0	0	w_4
1	0	1	1	w_5
1	0	1	0	w_6
1	0	0	1	w_7
1	0	0	0	w_8
0	1	1	1	u	...	v	$1 - uv$...	w_9
0	1	1	0	w_{10}
0	1	0	1	w_{11}
0	1	0	0	w_{12}
0	0	1	1	w_{13}
0	0	1	0	w_{14}
0	0	0	1	w_{15}
0	0	0	0	w_{16}

- No necessity (causality) but logical necessity (causality).

Carnap's Modal-Logical Space (Restricted)

p	q	r	s	ϕ	...	ψ	$\uparrow(\phi, \psi)$...		
1	1	1	1	x	...	y	$1 - xy$...	w_1	
	1	0	1	w_5	
»	1	0	1	0	w_6	
	0	1	1	1	u	...	v	$1 - uv$...	w_9
	0	1	0	1	w_{11}	
	0	0	1	1	w_{13}	
	0	0	1	0	w_{14}	
	0	0	0	1	w_{15}	
	0	0	0	0	w_{16}	

Rudolf Carnap (for instance) thinks, in w_6 , that there are necessities, e.g.,

- that bachelors are male by rule ($p \Rightarrow r$) and
- that men are rational by nature ($q \Rightarrow s$).

Generalized Possibilities

- Some propositions can be actually true while others are not.
- Some propositions can be possibly true while others can be not.
- A possibility is characterized by
 - (1) the atomic propositions that are true there;
 - (2) *and* what is its space of possibilities.

- (1) Which atomic propositions are true at a possibility w is defined by a valuation function V :

$V_w(p)$ is the value of a proposition p in any possibility w ;

- (2) The space of possibilities is given by an accessibility relation R :
deciding if a world w' is a possibility in a world w —
whereby $V_{w'}(p)$ decides whether p is true in w' , for any p ,
and, if so, whether such p are possible in w .

Modal Propositional Syntax

The language consists of formulas ϕ , defined in Backus-Naur style, relative to a set P of atomic proposition letters $p \in P$.

Syntax

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \rightarrow \phi) \mid \dots \mid \diamond\phi \mid \Box\phi$$

- [\diamond]: the weak, existential, modal operator;
- [\Box]: its strong, universal, counterpart.]

Modal Propositional Models

Kripke Model

A Kripke Model is a structure $\langle W, R, V \rangle$ with:

- W a non-empty set of worlds ('situations', 'states', 'indices'),
- R a relation on W ('accessibility'),
- V a valuation of the proposition letters in each world in W .

Pointed Kripke Model

A pointed Kripke Model is a structure $\langle W, R, V, w \rangle$ where

- $M = \langle W, R, V \rangle$ is a Kripke model,
- $w \in W$ is a designated point of evaluation.

[We also write ' M, w ' for ' $\langle W, R, V, w \rangle$ '.]

Modal Propositional Semantics

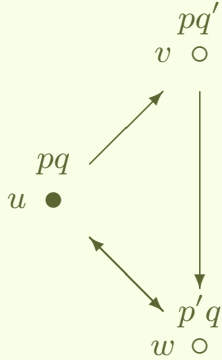
Semantics (for $M = \langle W, R, V \rangle$)

$M, w \models p$	iff $V(w)(p) = 1$;
$M, w \models \neg\phi$	iff $M, w \not\models \phi$;
$M, w \models (\phi \wedge \psi)$	iff $M, w \models \phi$ and $M, w \models \psi$;
$M, w \models (\phi \rightarrow \psi)$	iff $M, w \not\models \phi$ or $M, w \models \psi$;
...	...
$M, w \models \Diamond\phi$	iff there is w' : Rww' and $M, w' \models \phi$;
$M, w \models \Box\phi$	iff for all w' , if Rww' then $M, w' \models \phi$.

[Notation: $V_{M,w}(\phi) = 1$ (GAMUT) iff $M, w \models \phi$ (here).]

One (Pointed) Toy Model

[Legenda: Let \bullet be actual; “ p ” says that p is true where it is written, “ q' ” says that q is false where it is written.]



$$M = \langle \{u, v, w\}, R, V, u \rangle,$$

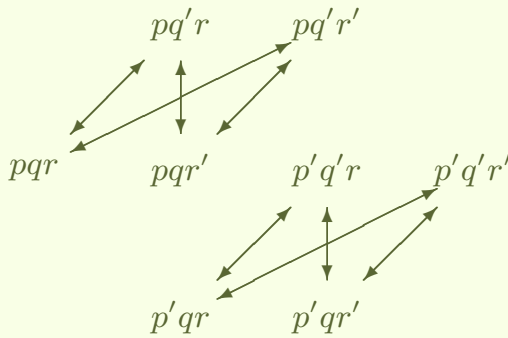
$$R = \{ \langle u, v \rangle, \langle u, w \rangle, \langle v, w \rangle, \langle w, u \rangle \},$$

$$V_u(p) = V_u(q) = V_v(p) = 1,$$

$$V_v(q) = V_w(p) = 0, V_w(q) = 1.$$

$M, u \models \Diamond p$	$M, u \models \Diamond \neg p$
$M, u \models \Diamond q$	$M, u \models \Diamond \neg q$
$M, u \models \neg \Diamond(p \wedge q)$	$M, u \models \Box(p \vee q)$
$M, u \models \Diamond \Diamond \Box(p \wedge q)$	$M, u \models \Diamond \Diamond \Box \neg(p \wedge q)$

Another (Non-Pointed) Toy Model



$$M \models (p \leftrightarrow \Box p), M \models (\neg p \leftrightarrow \Box \neg p)$$

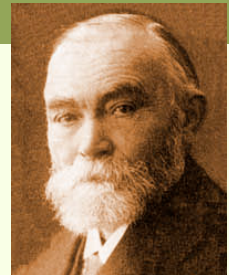
$$M \models (q \leftrightarrow \Box \neg q), M \models (\neg q \leftrightarrow \Box q)$$

$$M \models (\Diamond r \wedge \Diamond \neg r)$$

- Let $\Box :=$ “G believes that” and $\Diamond :=$ “G doesn’t exclude that.”
- G is right about p , but wrong about q , and doesn’t know about r .
- G is right about p and wrong about q and G knows it:
 - $M \models \Box(p \leftrightarrow \Box p)$ and $M \models \Box(q \leftrightarrow \Box \neg q)$, etc.
- There is something so not right, about the above **frame!**

GOTTLOB FREGE

Friedrich Ludwig Gottlob Frege



1848 – 1925, born in Wismar, works in Jena.

- *Begriffsschrift* (1879)
- *Grundlagen der Arithmetik* (1884)
- *Grundgesetze der Arithmetik* (1893)
 - instigates logicist program
 - invents modern logic
 - defines philosophy of language
- “Über Sinn und Bedeutung” (1892) • “der Gedanke” (1918) •

The conclusion appears to be this: Thoughts [Gedanke] are neither things from the external world, nor ‘imagination’ [Vorstellungen]. A third realm has to be acknowledged.” [Gottlob Frege, 1918, *Der Gedanke*, p. 69]

Platonism!

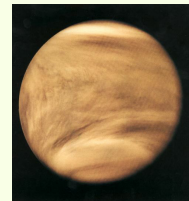
“Über Sinn und Bedeutung” (1892)

Die Gleichheit fordert das Nachdenken heraus durch Fragen, die sich daran knüpfen und nicht ganz leicht zu beantworten sind.

[“Identity gives rise to challenging questions which are not altogether easy to answer.” p. 25]

(11) “The Morningstar is the Eveningstar.”

(12) “The Eveningstar is the Eveningstar.”



- The first statement is ‘informative’ (it reports a discovery); the second is ‘analytic’ (its truth is a matter of meaning).
- If identity is a relation between objects this is not (yet) explained.
- Is identity therefore a relation between names? This cannot be.

The Signs are Arbitrary

If the sign “a” is distinguished from the sign “b” only as object (...), not as sign (i.e., not by the manner in which it designates something), the cognitive value of $a = a$ becomes essentially equal to that of $a = b$, provided that $a = b$ is true. (p. 26)

[That we also employ the form “a”—just like we employ the form “b”—to designate object b , does not teach us something about b .]

[It would supply conventional *linguistic* knowledge only.]

The *Bedeutung* of a Name is not its SINN

[Argument Reconstruction]

The meaning (here: SINN) of a whole is determined by the meanings (here: SINNE) of its parts, and their mode of composition.

(11) “The Morningstar is the Eveningstar.”

(12) “The Eveningstar is the Eveningstar.”

- The meaning (SINN) of sentence (11) changes, if we replace one term (“The Morningstar”) by another term (“The Eveningstar”) with the same reference (*Bedeutung*).

» Ergo, the reference (*Bedeutung*) of the two terms is not their meaning (SINN). [Exercise: why does this follow?]

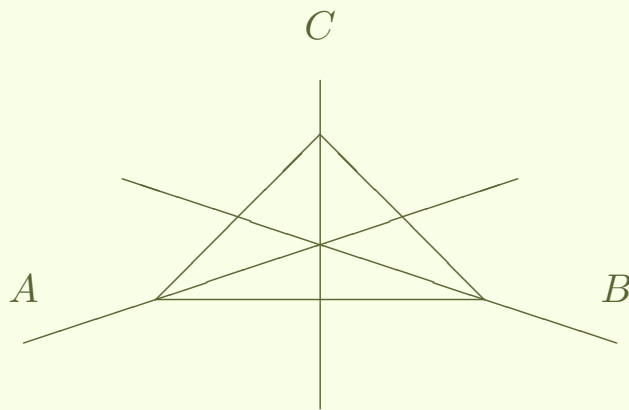
The SINN of a Name is the Mode of Presentation

A difference [in meaning] can arise only if the difference between the signs [Zeichen] corresponds to a difference in the mode of presentation of that which is designated. (...)

It is natural, now, to think of there being connected with a sign (...), besides that to which the sign refers, which may be called the referent of the sign [die Bedeutung des Zeichens], also what I would like to call the sense of the sign [den Sinn des Zeichens], wherein the mode of presentation [die Art des Gegebenseins] is contained. (p. 26)

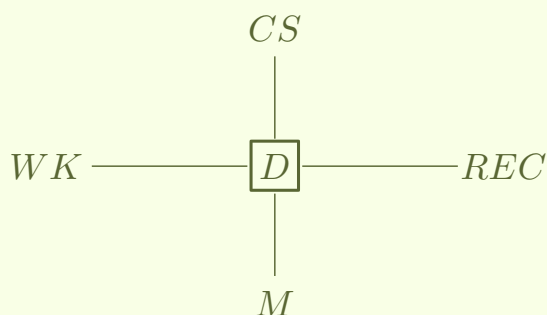
[Note: with the SINN, the mode of presentation, the *Bedeutung*, the object, if any, is given, or determined. Not the other way around!]

Frege's Triangle



- The intersection of the lines *A* and *B* (a point) **IS** the intersection of the lines *B* and *C* (a point), which is the intersection of the lines *A* and *C*.
- Three times, the *very same point* is the *Bedeutung*. Three times, it is *presented differently*, but objectively.

More Contingently: Dam Square



- The Square South of Central Station is the Square East of the Westerkerk that is the Square North of the Munt which is the Square West of the REC.
- Well, *that* is where we meet. Come to the *Bedeutung*.
- The “Erkenntnisswert” (*Sinn*) of the five(!) phrases is *not* the same.

“Drei Stufen von Verschiedenheit”

<i>Bedeutung</i>	(“Meaning”, “Referent”)	(very real)
<i>Sinn</i>	(“Meaning”, “Sense”)	(abstract, but objective)
<i>Vorstellung</i>	(“Idea”, “Conception”)	(private, mental)

A painter, a horseman, and a zoologist will probably connect different conceptions with the name “Bucephalus”. This constitutes an essential distinction between the conception and the sign’s sense, which may be the common property of many and therefore is not a part or a mode of the individual mind. (...) (p. 29)

Von den Vorstellungen und Anschauungen soll im folgenden nicht mehr die Rede sein; sie sind hier nur erwähnt worden, damit die Vorstellung, die ein Wort bei einem Hörer erweckt, nicht mit dessen Sinne oder dessen Bedeutung verwechselt werde. (p. 31)

[In what follows there will be no further discussion of conceptions and experiences; they have been mentioned here only to ensure that the conception aroused in the hearer by a word shall not be confused with its sense or its referent.]

Overview of *Über Sinn und Bedeutung*

- « Names have a sense (25–27);
 - Clarification of the relevant concepts (27–32);
- » Sentences have a reference (32–35);
 - Testing the hypothesis (35–36);
 - [“[D]ie Betrachtung der Nebensätze” (36–49)];
 - Conclusion (50).



FREGE (PART II)

A GEDANKE is the SINN of a Sentence

- Names have a reference (*Bedeutung*) that is not their meaning (SINN). The latter is the way in which the former is given.
- Every sentence is associated with a thought (Gedanke).
- Is this Gedanke the SINN of a sentence, or its *Bedeutung*?
- Surely the Gedanke is *not* the *Bedeutung*.

[Proof? Employ the observations on slide 31, using the principle of Compositionality of Reference (*Bedeutung*) this time.]

- Hence, the Gedanke constitutes its SINN.

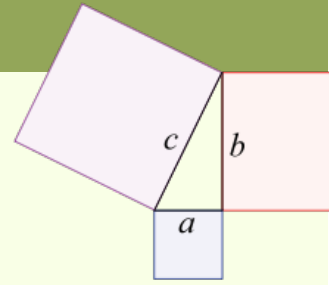
A GEDANKE is What is True or False

[Think, or read aloud]

$$a^2 + b^2 = c^2$$

Venus follows an elliptic orbit around the sun.

Circe has drugged Odysseus' men.



[Suppose you determine]

“That is false/true.”

“I have seen it.”

“I have heard it.”

“It has been proved.”

[Now wonder]

“What is it, that is false/true, proved, . . .?”

The Grasped Thought!

GEDANKE in Science

I am not here in the happy position of a mineralogist who shows his audience a rock-crystal: I cannot put a thought in the hands of my readers with the request that they should examine it from all sides. Something in itself not perceptible by sense, the thought, is presented to the reader — and I must be content with that — wrapped up in a perceptible linguistic form. [Frege, 1918, Der Gedanke, fn. 6]

The practice of science consists in the grasping of thoughts, the acknowledgement of their truth, or falsity, and the communication of your findings.

Pertinent to all of this is:

- An unchangeable thought.
- With a non-negotiable truth value.
- With inalienable logical consequences.

A Thought IS IS true/false.
↙ *a timeless “is”*

The *Bedeutung* of a Sentence is its Truth Value

- A sentence has a SINN, which is the thought expressed by it. Does it also have *Bedeutung*? Do we need a *Bedeutung*?
- Merely thinking a thought does not satisfy the scientist.

“Ulysses wurde tief schlafend in Ithaka ans Land gesetzt.”

Why are we not satisfied with the thought? Because, and as far as, we are concerned with its truth-value. (...) [W]henver the referents of its components are involved (...) this is the case when and only when we are inquiring after the truth value. (p. 33)

- The truth or falsity of the thought expressed is the *Bedeutung* of a sentence.

Sinn and Bedeutung of Sentences

- Just like names, also sentences have a SINN and a *Bedeutung*.
The SINN of the sentence is the thought expressed.
The *Bedeutung* of the sentence is the truth value of the thought.
“das Wahre”, if the thought presents the truth, or
“das Falsche”, if it doesn't.
- SINN as well as *Bedeutung* are objective.
- SINN is NOT *Vorstellung*: A *Vorstellung* is subjective, person-bound, time-bound, and a proper constituent of a particular consciousness; a SINN can be shared by many, across space, time, and generations.

For one cannot of course deny, that mankind possesses a treasure of thoughts, that it passes on from generation to generation. [p. 29]

The (Almost) Complete Picture

	Zeichen (sign)	Eigenname (name)	Satz (sentence)
drückt aus (expresses)	Sinn (sense)	Art des Gegebenseins (mode of presentation)	Gedanke (thought)
bezeichnet (refers to)	Bedeutung (reference)	Gegenstand (object)	Wahrheitswert (truth value)

- For “Begriffswort” (predicate), “Begriff” and “Funktion” (concept/function/set): see elsewhere.

Thesis Testing (p. 35–36)



- The reference $RE(\phi)$ of sentence ϕ is its truth value $TV(\phi)$, so:
 1. $RE([t/u]\phi) = RE(\phi)$ if $RE(t) = RE(u)$ entails
 $TV([t/u]\phi) = TV(\phi)$ if $RE(t) = RE(u)$.

Und das ist in der Tat der Fall. Leibniz erklärt geradezu: “Eadum sunt, quae sibi mutuo substitui possunt, salva veritate”. (p. 35)

- “And that is correct!” Quite remarkable!

Thesis Testing (p. 35–36)

- The reference $RE(\phi)$ of sentence ϕ is its truth value $TV(\phi)$, so:
 2. $RE([T/S]\phi) = RE(\phi)$ if $RE(T) = RE(S)$ entails
 $RE([T/S]\phi) = RE(\phi)$ if $TV(T) = TV(S)$.

Exceptions are to be expected when the whole or the constituent sentence are in direct or indirect mode; for (...) the words do not have their ordinary Bedeutung then. A sentence in direct speech refers to a sentence, and in indirect speech to a thought. (p. 36)

Indirect Modes, References and Senses

- In *direct quotation* expressions refer to themselves.
This is hopefully obvious. Is it?
- In *indirect quotation* expressions refer to their ordinary sense.
This extraordinary reference is given by an extraordinary sense, the mode of presentation of the ordinary sense.
- The *sense* of “Henry is in France”—i.e., the thought that Henry is in France—is the *referent* of that sentence in indirect use.
- When the ordinary sense is the extraordinary reference, it can be presented in various different ways.

Reference to Thoughts

- Charles believes that Henry is in France.
 - » That Henry is in France is what Charles believes.
- Suzanne doubts that Henry is in France.
 - » That Henry is in France is what Suzanne doubts.
- Charles believes what Suzanne doubts.
 - » What Charles believes = what Suzanne doubts.
- Charles believes what Charles believes.
 - » What Charles believes = what Charles believes.
- The Evening- / Morning-star problem is back again.
 - » Raised to a higher level.
- A Hierarchy of Senses unfolds.
 - » [Actually acknowledged by Frege himself, p.c.]

21-th Century Frege



[*Amy FF*]: *My colleagues and I are mapping the neurological substrates that subserve global information processing, which is required for all cognitive reasoning, including scientific inquiry, making my research ipso facto prior in the ordo cognoscendi. That means it's better than his research, and by extension, of course, yours.*

[*Sheldon C*]: *Excuse me, but a grand unified theory, insofar as it explains everything, will ipso facto explain neurobiology.*

[*Amy*]: *Yes, but if I'm successful, I will be able to map and reproduce your thought processes in deriving a grand unified theory, and therefore, subsume your conclusions under my paradigm.*

[*Sheldon*]: *That's the rankest psychologism, it was conclusively revealed as hogwash by GOTTLOB FREGE in the 1890s!*

[*The Big Bang Theory*, "The Zazzy Substitution", S4/e3, 7–10–2010
https://www.youtube.com/watch?v=V_TQea0a0nE]

WORLDS

A Critical Note As An Appetizer

In a truth-conditional semantics, the standard way of representing the status of situations [with beliefs, wants, wills, and if's] is as possible worlds: there is the real world, and then there are worlds with situations that are possible but not (necessarily) actual. Possible worlds are then identified with a person's beliefs or wishes or some other mental attitude. Possible worlds pose metaphysical problems for many people. Do possible worlds exist? If so – or especially if not – where are they?

[Croft&Cruse, 2004, *Cognitive Linguistics*, p. 33]

Possible Reply: They are everywhere where you turn your mind!

Ontological Stance on Possible Worlds (Michael Loux)

Michael J. Loux (ed.), 1979, *The Possible and the Actual*,
Cornell University Press, Ithaca.]

- Extreme Realists
- Die-hard Extensionalists
- “Moderate Realists”
- Instrumentalists

Gottfried Wilhelm Leibniz (1646—1716)

Diese Ursache muss überdies Verstand haben:

(...) denn da diese existierende Welt zufällig ist, und da eine Unendlichkeit von anderen Welten ebenso möglich ist und ebenso sehr wie sie (sozusagen) Anspruch auf Existenz macht, so muß die Ursache der Welt auf alle diese möglichen Welten Rücksicht genommen oder zu ihnen in Beziehung gestanden haben, damit sie für eine von ihnen entscheiden konnte.

... for since this existing world is incidental, and since an infinity of other worlds is equally possible, that just as like may claim existence (so to speak) ...

[G.W. Leibniz, *Studien zur Theodizee. Über die Güte Gottes, die Freiheit des Menschen und den Ursprung des Übels*, 1710, 1-er Zeil, §7]

[*The Extremist*]

Willard V.O. Quine (1908—2000)

The upshot of these reflections is meant to be that the way to do quantified modal logic, if at all, is to accept Aristotelian essentialism.

Such a philosophy is as unreasonable by my lights as it is by Carnap's or Lewis's. [C.I., PD, not D.K.] And in conclusion I say, as Carnap and Lewis have not: so much the worse for quantified modal logic. By implication, so much the worse for unquantified modal logic as well; for, if we do not propose to quantify across the necessity operator, the use of that operator ceases to have any clear advantage over merely quoting a sentence and saying that it is analytic.

[W.V.O. Quine, 1953, "Reference and Modality", p. 30–31]

[*The Nihilist*]

David Lewis (1941—2001)

I believe, and so do you, that things could have been different in countless ways. (...) Ordinary language permits the paraphrase: there are many ways things could have been besides the way they actually are. (...) It says that there exist many entities of a certain description, to wit 'ways things could have been'. (...) I believe permissible paraphrases of what I believe; (...) I therefore believe in the existence of entities that might be called 'ways things could have been'. I prefer to call them 'possible worlds'.

[David Lewis, 1973, *Counterfactuals*, Blackwell, Oxford, pp. 84–91; more in *On the Plurality of Worlds*, 1986, Blackwell, Oxford.]

[*The One Realist*]

Robert Stalnaker (1940—)

*The argument suggests not that ordinary language and our common beliefs commit us to a weighty metaphysical theory, but rather that what appears to be a weighty metaphysical theory is really just **some ordinary beliefs by another name**. Believing in possible worlds is like speaking prose. We have been doing it all our lives.*

[Robert Stalnaker, 1976, *Noûs*, 10, pp. 65–75; extended in *Inquiry*, 1984, Bradford Book, MIT, Cambridge MA.]

[*The Other Realist*]

Lewis on Ordinary Belief in Philosophy

- Belief in possible worlds is a natural belief.
- There is no sensible alternative to this belief. (Assgnmnt.)
- There is nothing wrong with the belief.

One comes to philosophy already endowed with a stock of opinions. It is not the business of philosophy either to undermine or to justify these preexisting opinions, to any great extent, but only to try to discover ways of expanding them into an orderly system.

[Lewis, *PW*, p. 88]

Actuality is Indexical

Our world is only one world among others. We call it alone actual not because it differs in kind from all the rest but because it is the world we inhabit.

The inhabitants of other times may truly call their own times ‘present’, if they mean by ‘present’ what we do.

[Lewis, *Possible Worlds*, p. 85–86]

- Actuality is not a specific property of the actual world but something purely indexical.
- If we believe in possible worlds we only believe in more things of the kind of stuff we already believe in: the actual world.
- Just systematic thinking about the possibilities we already know.

Lewis by way of Robert Stalnaker (1976)

(L1) Possible worlds exist.

[Only as a manner of (theoretically) speaking, says RS.]

(L2) They are of the same sort as the actual world.

(“I and all of my surroundings”)

[This is a *category mistake*, see below.]

(L3) Actuality is an indexical notion.

[This remark is MISPLACED, see below.]

(L4) Possible worlds are irreducible.

[But can be given a naturalistic explanation, says RS.]

“The World Is the Way It Is” (As Against L2)

“the way Bill is” $\rightsquigarrow \bigwedge \{P \mid P(b)\}$

Consider two readings of “Bill is the way he is”.

- $\bigwedge \{P \mid P(b)\}(b)$: Bill *has* the property that is the conjunction of all his properties. This is like “Bill is awesome”.

This is OK, and it is TRUE.

- $b = \bigwedge \{P \mid P(b)\}$: Bill *is* the property that is the conjunction of all his properties. This is like “Bill equals awesome”.

This is NONSENSE.

The way the world is
is not the same (kind of) thing as
the world that is that way.

Actual is Indexical, but Real (Qualifying L3)

Actuality seems to be (...) *an attribute which our world has relative to itself but which all other worlds have relative to themselves too.*

So, (...) *looking at things from an objective, absolute standpoint, merely possible people and their surroundings are just as real as we and ours?*

However, (...) *there is no such standpoint. The objective, absolute point of view is the view from within the actual world, and it is part of the concept of actuality that this should be so.*

Just as one could accept the indexical analysis of personal pronouns and be a solipsist, and accept the indexical analysis of tense and believe that the past exists only as memory and the future only as anticipation, one can accept the indexical analysis of actuality while excluding from one's ontology any universes that are the way things might have been.

And Solipsism and Presentism are Exclusive; Actualism is Inclusive.

Modal Realism

[A]s I believe that there really are other worlds, I am entitled to confess that there is much about them that I do not know, and that I do not know how to find out.

[Lewis, 1973, p. 88]

I am not bothered by the fact, freely admitted by Lewis, that there may be some modal facts—facts about what is possible—which we can never know. My worry is that I do not see how, on Lewis' account, there can be any other kind of modal fact.

[Stalnaker, 1984, *Inquiry*, p. 49]

GAMUT (1982 —), Etc.

*Possible worlds are convenient fictions
which help to make
a workable and perspicuous theory.*

[Speaks from GAMUT, e.g., pages 64 and 72.]

[*The Instrumentalists*]

FRAMES

Modal Propositional Logic

Syntax

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \rightarrow \phi) \mid \dots \mid \Diamond\phi \mid \Box\phi$$

Semantics

$$\langle W, R, V, w \rangle \models \phi \quad (\text{see } \mathcal{S}16)$$

Logic

Propositional logic plus:

$$\vdash \Diamond\phi \leftrightarrow \neg\Box\neg\phi \text{ and } \vdash \Box\phi \leftrightarrow \neg\Diamond\neg\phi \text{ (dualities)}$$

$$\vdash \Box\phi \text{ from } \vdash \phi \text{ (necessitation)}$$

$$\vdash \Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi) \text{ (distribution)}$$

Modal Propositional Logical Intuitions

What do we think about:

- $\Box\phi \rightarrow \phi?$
- $\Box\phi \rightarrow \Box\Box\phi?$
- $\Diamond\phi \rightarrow \Box\Diamond\phi?$

How *logical* are those?

Think of the various types of alternatives:

<i>formal</i>	<i>alethic</i>	<i>deontic</i>	<i>temporal</i>	<i>doxastic</i>
\Box	necessary	obliged	always	believe
\Diamond	possible	permitted	sometimes	not_exclude



Relativized Validities

$\langle W, R, V, w \rangle \models \phi$ (satisfaction on a pointed model)
(see slide 16)

$\langle W, R, V \rangle \models \phi$ (validity on a model)

ϕ is valid on a model $M = \langle W, R, V \rangle$ iff
 $\forall w \in W: \langle W, R, V, w \rangle \models \phi$

$\langle W, R \rangle \models \phi$ (validity on a frame)

Definition (Validity on a Frame)

ϕ is valid on a frame $F = \langle W, R \rangle$ iff
for all valuations $V: \langle W, R, V \rangle \models \phi$

$\models \phi$ (logical validity)
(standard)

Characterizing Frames

Certain metaphysical principles correspond to formal properties of frames that model the modal universe we think we inhabit.

Principle ϕ characterizes a property \mathcal{P} of frames \equiv
 ϕ is valid on a frame F iff F has property \mathcal{P} .

The formula (scheme or principle) $\Diamond p \leftrightarrow \Box p$, for instance, is valid on frames in which R is a function on W . How would that come about?

Philosophical and formal replies are feasible!



Necessities are True

$$\Box p \rightarrow p$$

- What is necessarily so, is true. (Philosopher's Assumption.)

*When something is true in all possibilities, it is true in actuality.
 Why would that be so?*

- Mundanely speaking, actuality is a possibility itself.

Modal space includes actual space.

- Formally speaking, for any w : wRw , i.e.:

R is a REFLEXIVE relation on W .

Reflexivity

$(\Box p \rightarrow p)$ is valid on $F = \langle W, R \rangle$ iff R is a reflexive relation on W .

(\Leftarrow) Suppose R IS a reflexive relation on W .

- If $\langle W, R, V, w \rangle \models \Box p$ in any $w \in W$ and with any V , then, since wRw (reflexivity), $\langle W, R, V, w \rangle \models p$.
- Hence $\langle W, R, V, w \rangle \models (\Box p \rightarrow p)$, for any $w \in W$ and any V .

! $F \models (\Box p \rightarrow p)$.

(\Rightarrow) Suppose R IS NOT a reflexive relation on W .

- So there is $w \in W$ such that not wRw .
- Let $V: V_w(p) = 0$ and $V_{w'}(p) = 1$ for all w' such that wRw' .
(This is possible because not wRw !)
- Then $\langle W, R, V, w \rangle \not\models (\Box p \rightarrow p)$.

! $F \not\models (\Box p \rightarrow p)$.

Necessities are Necessary

$\Box p \rightarrow \Box \Box p$

- Something really necessary is necessarily necessary.

If something holds in ALL possibilities, then it holds in ALL POSSIBLE possibilities. Where does that come from?

- Mundanely, all possible possibilities are possibilities.

Modal space does not expand.

- Formally, for any u, v, w : if uRv and vRw then uRw , i.e.:

R is a TRANSITIVE relation on W .

Transitivity

$(\Box p \rightarrow \Box\Box p)$ is valid on $\langle W, R \rangle$ iff R is transitive.

(\Leftarrow) Suppose R IS a transitive relation on W in F .

- Suppose₀ $\langle W, R, V, u \rangle \models \Box p$ in any $u \in W$ and with any V .
- Take arbitrary₁ $v: uRv$ and then arbitrary₂ $w: vRw$,
whence uRw (transitivity) and₀ $\langle W, R, V, w \rangle \models p$.
- Hence₂, $\langle W, R, V, v \rangle \models \Box p$ and₁ $\langle W, R, V, u \rangle \models \Box\Box p$.
- So $\langle W, R, V, u \rangle \models (\Box p \rightarrow \Box\Box p)$, for any $u \in W$ and any V .

! $F \models (\Box p \rightarrow \Box\Box p)$.

(\Rightarrow) Suppose R IS NOT a transitive relation on W .

- So there are $u, v, w \in W$ such that uRv and vRw but not uRw .
- Let $V: V_w(p) = 0$ and $V_{u'}(p) = 1$ for all u' such that uRu' .
(This is possible because not uRw !)
- Then $\langle W, R, V, u \rangle \not\models (\Box p \rightarrow \Box\Box p)$.

! $F \not\models (\Box p \rightarrow \Box\Box p)$.

Possibilities are Necessary

$\Diamond p \rightarrow \Box\Diamond p$

- Something possible is impossibly impossible.

What is possible must of necessity be possible. How so?

- Every possibility is a possibility in all possibilities.

Modal space does never shrink.

- For any u, v, w : if uRv and uRw then wRv , i.e.:

R is EUCLIDEAN.

From K to $S5$

Let $[u]_R = \{v \mid uRv\}$ be the modal space of u .

We find that $u \in [u]_R$ if R is REFLEXIVE.

If uRw then $[u]_R \supseteq [w]_R$ if R is TRANSITIVE.

If uRw then $[u]_R \subseteq [w]_R$ if R is EUCLIDEAN.

If R is REFLEXIVE, TRANSITIVE and EUCLIDEAN then R PARTITIONS logical space W in a set of modal universes $\{[u]_R \mid u \in W\}$.

- In this system ($S5$) all stacks of modalities reduce to a simple one:

$$\diamond\Box\phi \Leftrightarrow \Box\phi \Leftrightarrow \Box\Box\phi \text{ and } \diamond\diamond\phi \Leftrightarrow \diamond\phi \Leftrightarrow \Box\diamond\phi.$$

From Actuality to Possibility

$$p \rightarrow \diamond p$$

- Something actual is, of course, possible.
- So, whenever something is true in actuality, it is true in a possibility. How can that be?
- Actuality is a possibility itself. So, for any w : wRw . R is a reflexive relation on W .
- Didn't we see that before?

Necessity and Impossibility

- Do $(\Box p \rightarrow p)$ and $(p \rightarrow \Diamond p)$ say the same thing?
- They are surely not equivalent. However,
 - $(\Box p \rightarrow p)$ and $(\neg p \rightarrow \Diamond \neg p)$ are equivalent.
 So $(\Box p \rightarrow p)$ and $(p \rightarrow \Diamond p)$ are equivalent *principles*.
- So they characterize the same frame property (reflexivity).
- Likewise, the same principles (frame properties) are expressed by:
 - $(\Box p \rightarrow \Box \Box p)$ and $(\Diamond \Diamond p \rightarrow \Diamond p)$ (transitivity),
 - $(\Diamond p \rightarrow \Box \Diamond p)$ and $(\Diamond \Box p \rightarrow \Box p)$ (euclidicity),
 - $(p \rightarrow \Box \Diamond p)$ and $(\Diamond \Box p \rightarrow p)$ (symmetry).
- The latter is Intriguing (See Hughes and Cresswell, footnote p. 70, on the label “Brouwersche” axiom.)

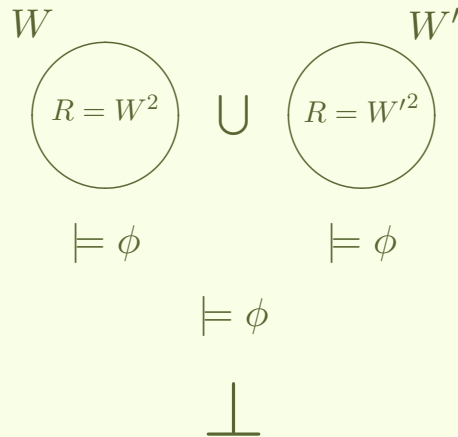
Unknownables

- How can we possibly speak of possibilities we cannot speak of?
 - We cannot. (Ludwig Wittgenstein)
- No formula (principle) guarantees that all possibilities are accessible, i.e., that R is a total relation on a frame.
- Tough? Irreflexivity cannot be characterized either!
- Heavy? Or just Cool? (“Is the world I see the world I see?”)

It is COOL.

Illustration for next Slide

» Suppose ϕ characterizes universality.



A Formal Result with a Philosophical Ring (1)

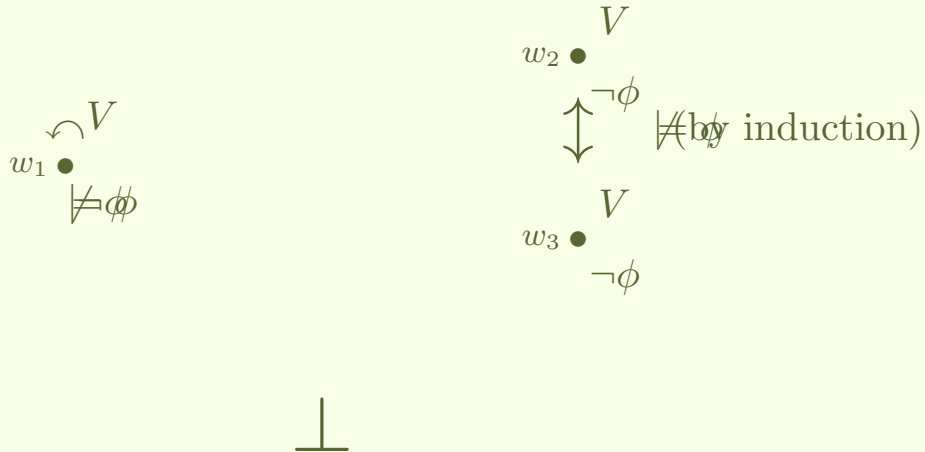
» Suppose ϕ characterizes universality.

- Let $F = \langle W, R \rangle$ and $R = W^2$. R is universal, so ϕ is valid on F , whence $\langle W, R, V, w \rangle \models \phi$ for any $w \in W$ and valuation V .
- Duplicate F into a disjoint $F' = \langle W', R' \rangle$ with $R' = W'^2$. R' is also universal, so $\langle W', R', V', w' \rangle \models \phi$ for any $w' \in W'$ and V' .
- Take the union $F'' = \langle W'', R'' \rangle$ of the two frames, so $W'' = (W \cup W')$ and $R'' = (R \cup R')$.
- Any V'' for F'' equals the union of some V and V' for F and F' .
- For any such V'' , and for any $w'' \in W''$: $\langle W'', R'', V'', w'' \rangle \models \phi$.
- Hence, the frame $F'' = \langle W'', R'' \rangle \models \phi$.
- But surely R'' is not a universal relation on W'' .
- Contradiction.

« So there is no such formula ϕ . [I.e., even if you believe in S5, you cannot exclude the existence of other modal universes.] [But, who cares?]

Illustration for next Slide

» Suppose ϕ characterizes irreflexivity.



A Formal Result with a Philosophical Ring (2)

» Suppose ϕ characterizes irreflexivity.

- Let $F = \langle W, R \rangle$ with $W = \{w_1\}$ and $R = \{\langle w_1, w_1 \rangle\}$.
- Let $F' = \langle W', R' \rangle$ with $W' = \{w_2, w_3\}$ and $R' = \{\langle w_2, w_3 \rangle, \langle w_3, w_2 \rangle\}$.
- F is not irreflexive, so for some V : $\langle W, R, V, w_1 \rangle \not\models \phi$.
- For all V for F there is V' for F' such that $V_{w_1} = V'_{w_2} = V'_{w_3}$.
- $\langle W, R, V, w_1 \rangle \models \psi$ iff $\langle W', R', V', w_2 \rangle \models \psi$ iff $\langle W', R', V', w_3 \rangle \models \psi$.
 - This can be shown by a straightforward induction on ψ .
- So $\langle W', R', V', w_2 \rangle \not\models \phi$ and $\langle W', R', V', w_3 \rangle \not\models \phi$.
- But F' is irreflexive, whence ϕ should be valid on F' !
- Contradiction.

« So there is no such formula ϕ .

[No formula can tell two indistinguishable worlds apart. So no formula can decide whether those two (only) see each other, or (only) see themselves.]

TIMES

Arthur Prior (1914—1969)



Philosophy, including Logic, is not primarily about language, but about the real world. [F]ormal logic and general philosophy have more to bring to one another than is sometimes supposed.

[Prior, cited by Copeland, 2007, p. 1/21]

The formation-rules of the calculus of tenses are not only a prelude to deduction but a stop to metaphysical superstition.

[A. Prior, 1967, p. 19]

Two Impressions (Series) of Time

A Time that passes by.

$$\text{REMPAST}(e) \leftarrow \text{PAST}(e) \xleftarrow{we} \text{PRES}(e) \leftarrow \text{FUT}(e) \leftarrow \text{REMFUT}(e)$$

Oh my God, the exam is coming up. Thank goodness, that's over.

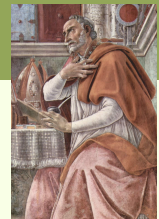
B Time that we move through.

$$we_{i-1} \rightarrow \dots \rightarrow we_i \rightarrow \dots \rightarrow we_{i+1}$$

BIGBANG e APOCALYPS

Tuesday we prepared for the exam. Thursday we celebrated we passed.

Prior's Confession



*Since men foresee the future and recall the past, and 'that which is not, cannot be seen', Augustine is tempted to say that even past and future events and moments in some sense 'are', and that there is some 'secret place' from which they come and to which they go. But (...) wherever 'time past and time to come' may 'be', 'they are not there as future, or as past, but present. For if there also they be future, they are not yet there; if there also they be past, they are no longer. **Wheresoever then is whatsoever is, it is only as present.**'*

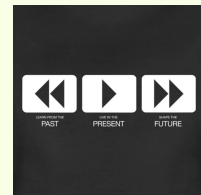
[A. Prior, 1967, *Past, Present and Future*, Clarendon Press, Oxford, p. 9]

Tenses as Temporal Operators

$S \Leftrightarrow S \text{ present,}$
 so, $S \text{ future} \Leftrightarrow (S \text{ present}) \text{ future,}$
 and, $S \text{ future} \Leftrightarrow (S \text{ future}) \text{ present.}$

Tenses are operators on propositions, like truth-functions and modal operators, and they have their own characteristic logic.

- (\diamond) F : somewhere in the **F**uture;
 P : somewhere in the **P**ast;
- (\square) G : always **G**oing to be in the future;
 H : always **H**as been in the past.



Tenses with Models

- **Temporal logic** is backed up by an (indexical) model-theory of time.
- **Tensed interpretation** is realized on a line (series) of dates with a distinguished point for *now*.
- **Prior's translation** into the l - ('later then') calculus, employs variables x for dates, and a distinguished variable z for 'now'.
- **Kripke models** avant la lettre, with 'standard translations' into FO logic. (Prior and Carew Meredith, 1956)

[A -series interpretation in a B -series model ;-]

Priorian Logic Now

- A tense logical model is a model $M = \langle W, R, V \rangle$ of modal logic, with $W = T$ a set of time instants and $R = <$ is ‘temporal precedence’. (Unconstrained, to begin with!)

$M, t \models F\phi$ iff $M, t' \models \phi$ at some instant t' : $t < t'$;

$M, t \models P\phi$ iff $M, t' \models \phi$ at some instant t' : $t' < t$.

$M, t \models G\phi$ iff $M, t' \models \phi$ at all instants t' : $t < t'$;

$M, t \models H\phi$ iff $M, t' \models \phi$ at all instants t' : $t' < t$.

- Just a modal logic, with two modalities, back- and forward-looking.
- The use of ‘<’ suggests an order, but it is just a relation, so far.

Basic Laws

- Dualities.

$\models G\phi \leftrightarrow \neg F\neg\phi$ (so $F\phi \leftrightarrow \neg G\neg\phi$),

$\models H\phi \leftrightarrow \neg P\neg\phi$ (so $P\phi \leftrightarrow \neg H\neg\phi$).

- Interaction Rules.

$\models (p \rightarrow GPp)$ (in the future the present is past),

$\models (p \rightarrow HFp)$ (the present is future of the past).

Future and past are each others mirror image.

- A-Priorean observations!



Indexical Principles

$$(p \rightarrow GPp)$$

(In the future the present will be past.)

\models Logically valid.

$$(Pp \rightarrow GPp)$$

(The past does not change in the future.)

$\not\models$ Not *logically* valid.

Constraining Temporal Precedence



- < may be transitive (if $t < t' < t''$ then $t < t''$);
 - < may be irreflexive ($t \not< t$);
 - < may be complete (if $t \neq t'$ then $t < t'$ or $t' < t$);
 - < may be linear (transitive, irreflexive and complete);
 - < may be dense (if $t < t''$ then $t < t' < t''$, for some t');
 - < may be asymmetric (if $t < t'$ then $t' \not< t$);
 - < may be circular ($t < \dots < t$);
 - < may be (un-)ending (there is no (some) t' : $t < t'$);
 - < may be non-branching (there are no $t < \{t', t''\}$: with $t' \not< t'' \not< t'$).
- Can we rule this by means of logical principles?
 - A whole lot of exercises!

Expressiveness of the Logic

Transitivity of the temporal order can be expressed in various ways.

$(PPp \rightarrow Pp)$, likewise: $(Hp \rightarrow HHp)$; similarly
 $(FFp \rightarrow Fp)$, likewise: $(Gp \rightarrow GGp)$.

What was past yesterday is past today. (That's part of the meaning of 'past'.) What always remains true will not suddenly no longer remain true. (That's part of the meaning of 'always'.)

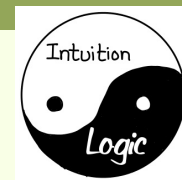
Even more elegantly.

$(Pp \rightarrow GPp)$.

Past is persistent. It is historically and politically incorrect to think you can change the past.

Yet all these *principles* say the same thing.

Limitations of (Our?) Temporal Logic

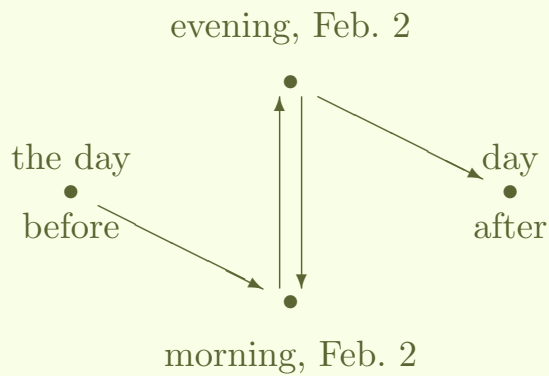


- Universality and irreflexivity cannot be characterized.
- But isn't that very intuitive, in, e.g., a temporal logic?
- Isn't time possibly circular?
- Can we exclude the possibility of parallel times?

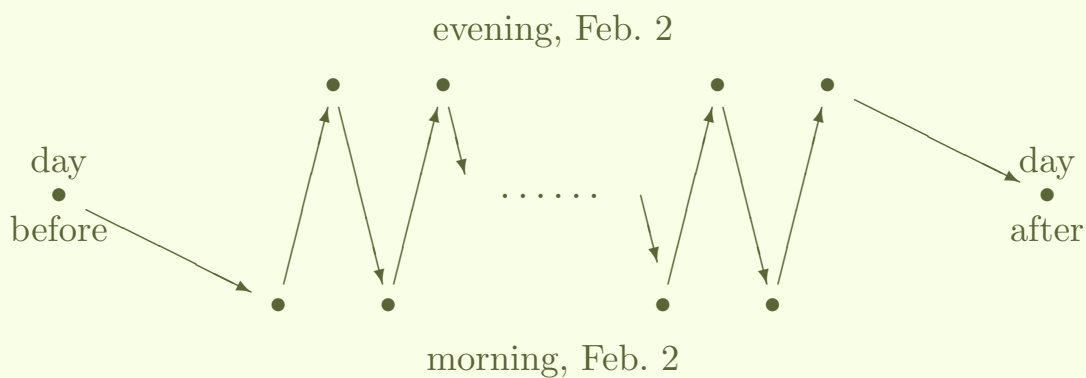
Groundhog Day, The Suggested Model



- February 2, 1993, Punxsutawney, Pennsylvania.



Groundhog Day, The Real Model



It is estimated that that one day lasted almost 34 years.

[Archived Reference.]

[A critical note on the concept of people returning from travels through time.]

Back from the Future

- Three days from now (next Monday) I have sent you back to now.
- You don't notice a difference, because there isn't any.
- If you go back to some time in the past, then everything there will be the same as it was then, otherwise it would not be the real past.
- Since you didn't know back then that you had then been sent back from the future, likewise you don't know now that you have now been sent back from the future.
- Until I tell you — that you have been. Now you know — again.

Since you are unable to distinguish the situation in which I actually do this from the one in which I do not, you have nothing to exclude either case.

- So you cannot have any ground for believing that I do/did not do so.

Practical Limitations of Priorean Logic



- Reference to times and intervals.
- Relations between times and intervals.
- Quantification over times and intervals.
- Iterated substructure of times and intervals.

(16) Asa was writing her PhD then.

(17) Ere grew an adult when (...)

(18) (...) Idi climbed the Ben Nevis.

(19) Ono won the Champions trophy several times.

(20) Ulu played the “Minute Waltz” for one hour without stopping.